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ANALYSIS OF WEBS IN DIAGONAL TENSION

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1.INTRODUCTION

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•The development of diagonal-tension webs is one of the most outstanding examples of departures of aeronautical design from the beaten paths of structural engineering.

•Standard structural practice had been to assume that the load bearing capacity of a shear web was exhausted when the web buckled.

•Prof. Herbert Wagner demonstrated that a thin web with transverse stiffeners does not “fail” when it buckles – it merely forms diagonal folds and functions as a series of tension diagonals, while stiffeners act as compression posts.

•The web-stiffener system thus functions like a truss and is capable of carrying loads many times greater than those producing buckling of the web.

•Use of diagonal-tension webs in the design of aero structural components has led to significant reduction in weight.

1.1 EXAMPLES OF DIAGONAL TENSION

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EXAMPLES OF DIAGONAL TENSION (contd)

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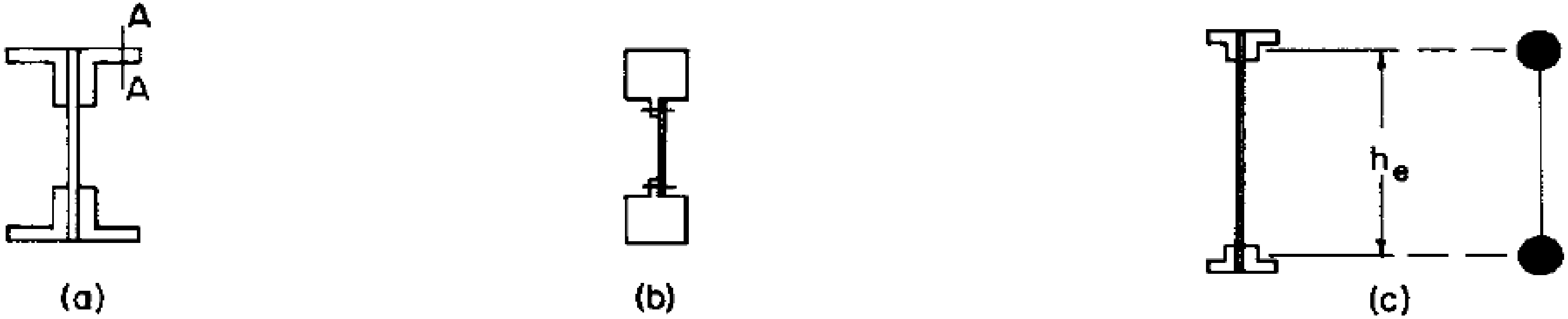
2.THEORY OF SHEAR RESISTANT BEAMS



Cross sections of built-up beams.

Figure 1

- Typical cross sections of built-up beams are shown in figure. The web is sufficiently thick to resist buckling up to failing load with or without the aid of stiffeners.
- Hence they are called “Shear buckling resistant” or for the sake of brevity “Shear resistant” beams.
- If the web to flange connections are adequately stiff, the stress in the built-up beams follow fairly well the formulas of the engineering theory of bending.

$$\sigma = \frac{M.z}{I} \qquad q = \frac{S.Q}{I}$$

Where σ = bending stress, q = shear flow, M = bending moment, Q = static moment of area about neutral axis, I = moment of inertia, z = distance of fiber from neutral axis.

2.1 SHEAR RESISTANT BEAMS (contd.)

•The shear flow distribution in the web of the beam subjected to a lateral shear force follows a parabolic law. In most cases the difference between the maximum (along the neutral axis) and minimum (along the flange rivet line) shear flow in the web is rather small and the design of the web is based on the average shear flow.

$$q_{av} = \frac{S.Q_F}{I}\left(1 + \frac{2Q_W}{3Q_F}\right)$$

Where q_{av} = average shear flow in web, Q_F = static moment about neutral axis of the flange area
Q_W = static moment of the web material above the neutral axis.

•When the depth of the flange is small compared with the depth of the beam (fig.1c) and the bending stresses in the web are neglected, the formulas are simplified to the so called "Plate-Girder Equations".

$$\boxed{\sigma_F = \frac{M}{h_e.A_F}} \longrightarrow \textbf{Eqn. 2a}$$

$$\boxed{q = \frac{S}{h_e}} \longrightarrow \textbf{Eqn.2b}$$

•However use of Plate-Girder equations give large errors if above mentioned assumptions are not valid. When the dimensions of the cross sections are extreme, the web to flange connections, particularly if riveted, is often overloaded and yields at low loads. The beam no longer acts as an integral unit, the two flanges tend to act as individual beams restrained by the web.

3.THEORY OF PURE DIAGONAL TENSION

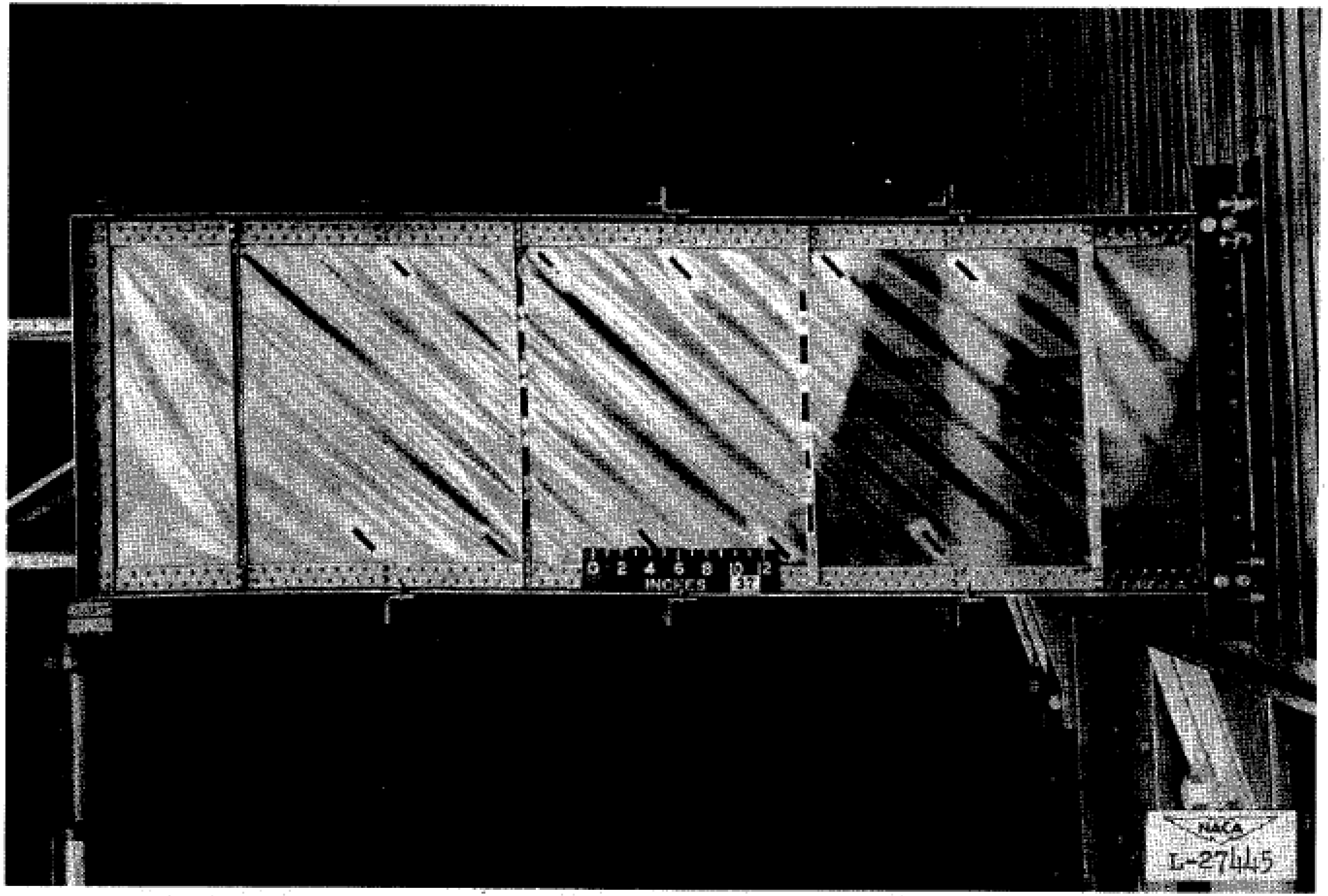


Figure 2 : Diagonal Tension Beam

3.1 BASIC CONCEPTS OF PURE DIAGONAL TENSION

•A diagonal-tension beam is defined as a built-up beam similar in construction to a plate-girder but with a web so thin that it buckles into diagonal folds at a load well below the design load (Figure 2).

•A Pure Diagonal Tension beam is a theoretical limiting stage in which the buckling of the web takes place at an infinitesimally small load. Practical structures only approach this limiting condition asymptotically.

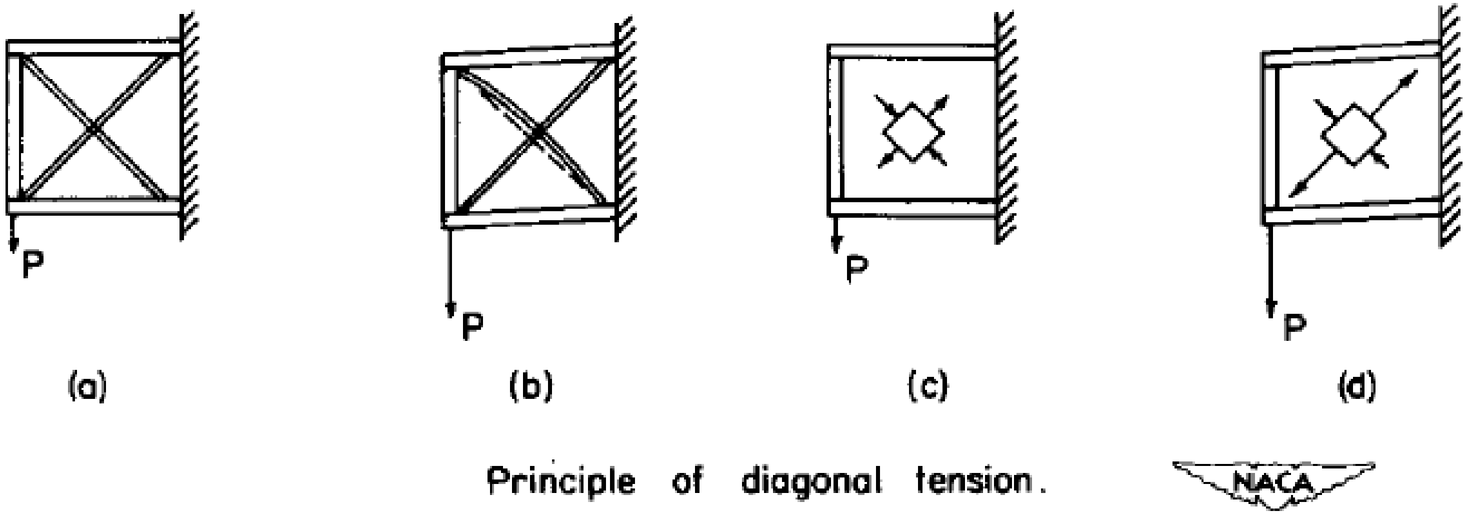


Figure 3

•The principle of diagonal tension can be understood by considering the structure shown in Figure 3 consisting of a parallelogram frame of stiff bars, hinged at the corners and braced internally by two slender diagonals of equal size.

3.2 BASIC CONCEPTS OF PURE DIAGONAL TENSION (Contd)

•As long as the applied load P is very small, the two diagonals will carry equal and opposite stresses.

•At a certain value of P, the compression diagonal will buckle (Fig 3b) and thus lose its ability to take additional large increments of stress.

•Consequently if P is increased further by large amounts, the additional diagonal bracing force must be furnished mostly by the tension diagonal.

•At very high applied loads, the stress in the tension diagonal will be so large that the stress in the compression diagonal is negligible by comparison.

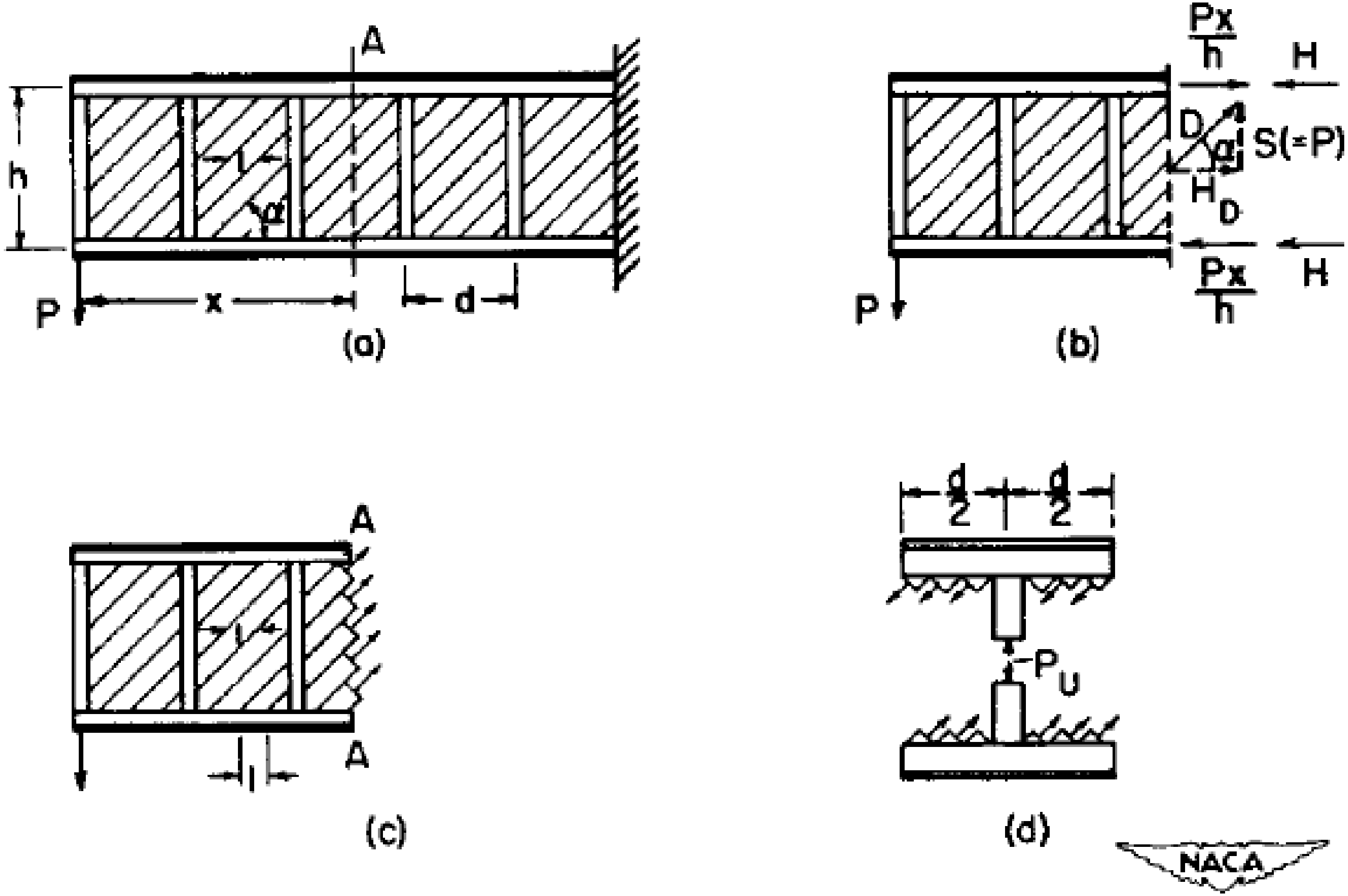
•An analogous change in the state of stress will occur in a similar frame in which the internal bracing consists of a thin sheet (Fig 3c). At low values of applied load, the sheet is in a state of pure shear, which is statically equivalent to equal tensile & compressive stresses at 45 degrees to the frame axes.

•At a certain critical value of the load, P, the sheet buckles, and as the load P is increased beyond the critical value, the tensile stresses become rapidly predominant over the compressive stresses (Fig 3d).

•The buckles develop a regular pattern of diagonal folds, inclined at an angle α and following the lines of the diagonal tensile stress.

•The tensile stress is so large that the compressive stress can be neglected entirely by comparison, the sheet is said to be in a state of **fully developed** or ***"pure"* diagonal tension.**

3.3 THEORY OF PRIMARY STRESSES. SASTRA



Forces in diagonal - tension beam .

Figure 4

3.4 THEORY OF PRIMARY STRESSES. (contd...)

- A girder with a web in pure diagonal tension is shown in figure 4(a). To physically define this condition, we assume that the web is cut into a series of ribbons or strips of unit width, measured horizontally. Each one of these strips is inclined at the angle α to the horizontal axis and is under a uniform tensile stress σ.

- The free body diagram as shown in figure 4(b) shows the internal forces in the strips intercepted by the section A-A. Since all the strips have the same stress, the resultant is located at mid-height.

- The horizontal component H_D of the resultant of the internal forces (D) is balanced by the compressive forces H in the two flanges. Hence, we have :

$$H_D = S.\cot\alpha \qquad \boxed{H = -\frac{S}{2}.\cot\alpha} \longrightarrow \textbf{Eqn. 3a}$$

- Computing the total flange forces we have :

$$F = \pm\frac{M}{h} + H \qquad F = \pm\frac{M}{h} - \frac{S}{2}.\cot\alpha$$

- With reference to figure 4 (c), each strip is cut at right angles, giving the stress carrying face a width of **1 x sin(α)**. The force on each strip is therefore **σ.t sin α**. The number of strips intercepted by section A-A is equal to **h cot α**. Therefore, the total force D on all the strips is **:**

$$D = \sigma.t.\sin\alpha \times h.\cot\alpha \qquad D = \sigma ht.\cos\alpha$$

3.5 THEORY OF PRIMARY STRESSES. (contd…)

•But from statics, we have : $D = \dfrac{S}{\sin\alpha}$

Therefore, $\dfrac{S}{\sin\alpha} = \sigma ht.\cos\alpha$

$$\boxed{\Rightarrow \sigma = \frac{2S}{ht.\sin 2\alpha}} \longrightarrow \textbf{Eqn. 3b}$$

• The upright is under compression counteracting the tendency of the diagonal tension to pull the flanges together (Fig 4(d)). The force P_U acting on each upright consists of the vertical components of the forces acting in all the strips appertaining to each upright, i.e. in "d"strips (since the strips have unit strips horizontally).

• The vertical component of h.cot α strips is equal to S (**ref Fig. 4(b)**) and hence by proportionality, we have :

$$P_U : S :: d : h\cot\alpha$$

$$\boxed{\Rightarrow P_U = -S\frac{d}{h}\tan\alpha} \longrightarrow \textbf{Eqn. 3c}$$

3.6 THEORY OF PRIMARY STRESSES. (contd…)

- If each strip is connected to the flange by one rivet, the force on this rivet is equal to the force σ.t.sinα in the strip. Since the strips are of equal width horizontally, rivet force per inch run R” can be given by :

$$R'' = \frac{S}{h.\cos\alpha}$$

- All the primary stresses having been expressed in terms of the knowns, namely the load P and the dimensions h and d; to complete the solution the angle α must be found. To compute α, the principle of least work may be used.

- The internal work in one bay of the beam is given by the expression :

$$W = \frac{\sigma^2}{2E}.dht + \frac{\sigma^2{}_U}{2E}.A_{Ue}h + \frac{\sigma^2{}_F}{2E}.A_F d$$

Note: A_{ue} indicates single uprights; all other terms are in usual notations.

3.7 THEORY OF PRIMARY STRESSES. (contd…)

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• By substituting into this expression, the stress values that follow from equations 3a, 3b and 3c which are :

$$\boxed{\sigma = \frac{2S}{ht\sin\alpha} = \frac{2\tau}{\sin 2\alpha}} \longrightarrow \textbf{Eqn. 3d}$$

$$\boxed{\sigma_U = -\frac{S.d}{hA_{Ue}}.\tan\alpha = -\frac{\tau dt}{A_{Ue}}.\tan\alpha} \longrightarrow \textbf{Eqn. 3e}$$

$$\boxed{\sigma_F = -\frac{S}{2A_F}.\cot\alpha = -\frac{\tau ht}{2A_F}.\cot\alpha} \longrightarrow \textbf{Eqn. 3f}$$

• Differentiating to obtain the minimum, and omitting the constant factor S^2/E, there results :

$$\frac{dW}{d\alpha} = -\frac{8d}{ht}.\frac{\cos 2\alpha}{\sin^2\alpha} + \frac{d^2}{hA_{Ue}}.\frac{\sin\alpha}{\cos^3\alpha} - \frac{d}{2A_F}.\frac{\cos\alpha}{\sin^3\alpha}$$

•Substituting into this equation the values for stresses given by equations 3d, 3e and 3f and equating to zero results in the relation :

$$-\sigma.\frac{4.\cos 2\alpha}{\sin^2\alpha} - \frac{\sigma_U}{\cos^2\alpha} + \frac{\sigma_F}{\sin^2\alpha} = 0$$

From which :

$$\boxed{\tan^2\alpha = \frac{\sigma - \sigma_F}{\sigma - \sigma_U}} \longrightarrow \textbf{Eqn. 3g}$$

3.8 THEORY OF PRIMARY STRESSES. (contd….)

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- Expressing σ, σ_F and σ_U in terms of S and α, we obtain :

$$\tan^4 \alpha = \frac{1 + \dfrac{ht}{2A_F}}{1 + \dfrac{dt}{A_{Ue}}}$$

Eqn. 3h

•Thus, by the use of equation 3h in equations 3d to 3f, we can obtain the stresses in terms of the applied shear force S and dimensions of the panel.

•In plane webs, the angle α generally does not deviate more than a few degrees from the average value of 40^o.

3.9 SECONDARY STRESSES IN PURE DIAGONAL TENSION

Secondary actions in diagonal – tension beams

Figure 5

•**Equations 3d, 3e and 3f** define the primary state of stress caused directly by diagonal tension. However there are also secondary stresses which should be taken into account when necessary.

•The vertical component of the **web stresses σ** acting on the flanges cause bending of the flanges between uprights as shown in Fig (5a). Considering the flange as a continuous beam supported by the uprights we have :

-The total bending load in one bay is equal to $\mathbf{P_U}$.

-Assuming this bending load to be uniformly distributed, the primary bending moment occurs at the uprights and is

$$M_{\max} = \frac{Sd^2 \tan\alpha}{12h}$$

Note that in the middle of the bay there is a secondary maximum moment half as large as the primary moment.

3.10 SECONDARY STRESSES IN PURE DIAGONAL TENSION (contd ..)

SASTRA

•**Fig (5a)** shows the deflections of the flanges when the bending stiffness of the flanges is small and these deflections sufficient to relieve the diagonal tension stress in those diagonal strips that are attached to the flange near the middle of the bay.

•The diagonals attached near the uprights must make up for this deficiency in stress and thus carry higher stresses than computed on the assumption that all diagonals are equally loaded (**Ref Fig (5b)**).

•This redistribution of the web tension stresses cause a reduction in the secondary flange bending moments. Wagner has proposed the following formulas, on the basis of simplifying assumptions to account for this effect :

$$\sigma_{\max} = (1 + C_2)\frac{2S}{ht \sin 2\alpha} \qquad M_{\max} = C_3 \frac{Sd^2 \tan\alpha}{12h}$$

Graphs for the factors C_2 and C_3 are given in **Section 4** of **Ref [1].**

•Factor C_2 and C_3 are functions of the flange –flexibility parameter ωd, which is defined by

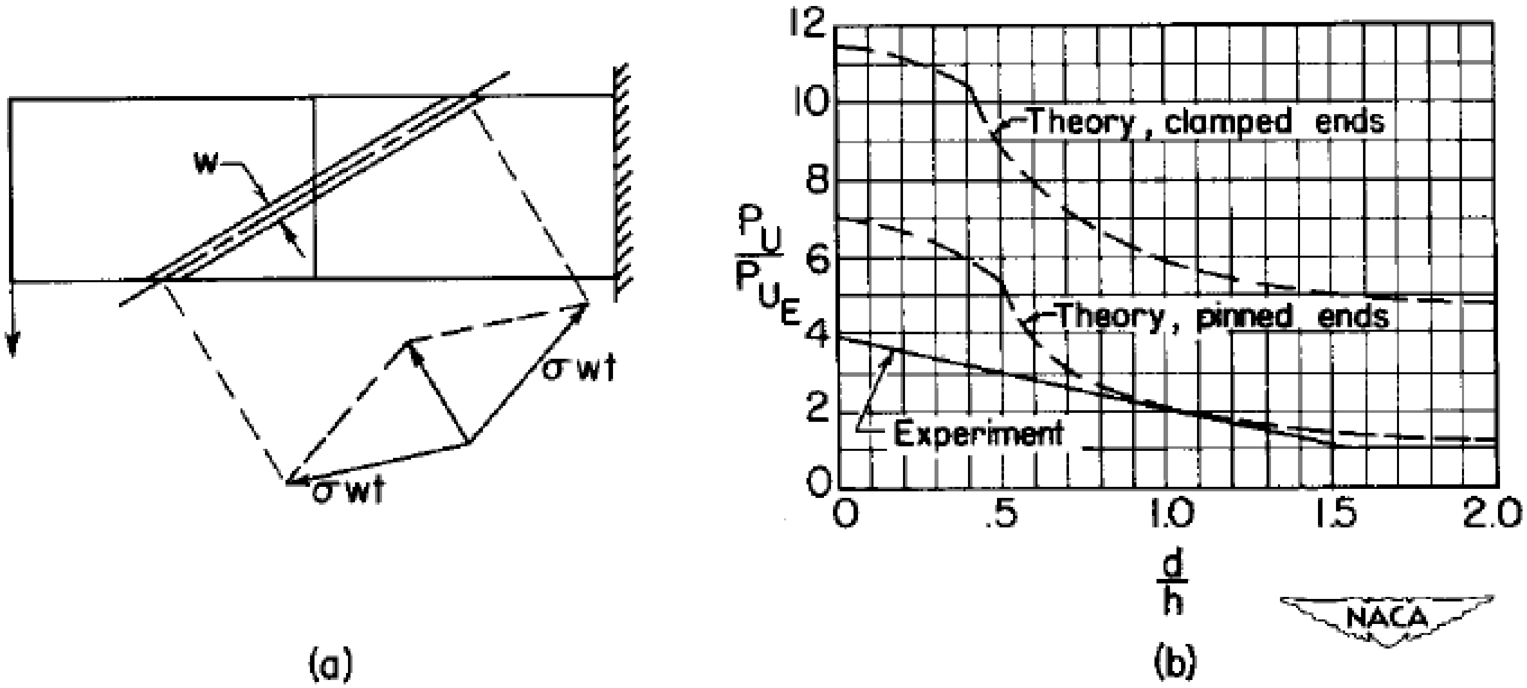
$$\boxed{\omega d = d \sin\alpha . \sqrt[4]{\left(\frac{1}{I_T} + \frac{1}{I_C}\right)\frac{t}{4h}}} \longrightarrow \textbf{Eqn. 3i}$$

Where subscripts T and C denote tension and compression flanges respectively.

3.11 BEHAVIOR OF UPRIGHTS IN PURE DIAGONAL TENSION

•The buckling strength of the uprights, be it single or double (on both sides of the web) cannot be calculated by ordinary column formulas as the web (to which the uprights are fastened) restrains the uprights against buckling.

•As soon as the upright begins to buckle out of the plane of the web, the tension diagonals crossing the upright becomes kinked at the upright, and the tensile forces in the diagonals develop components normal to the web tending to force the upright back into the plane of the web (Ref Fig (6a)).



Effect of diagonal tension on column length of uprights.

Figure 6

3.12 BEHAVIOR OF UPRIGHTS IN PURE DIAGONAL TENSION (contd..)

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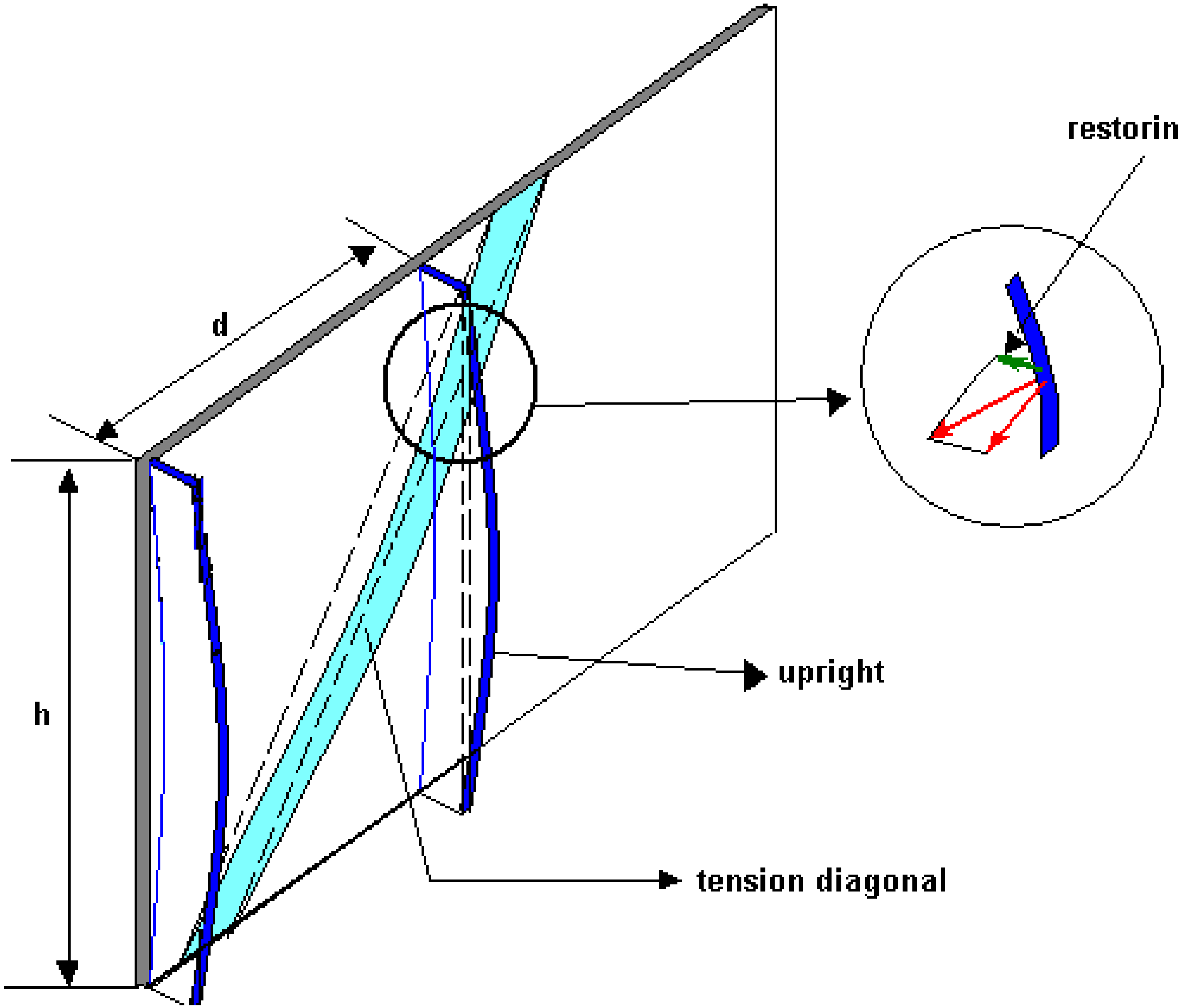

3.13 BEHAVIOR OF UPRIGHTS IN PURE DIAGONAL TENSION (contd..)

- The restoring force exerted by the diagonal tension band upon the upright is proportional to the deflection (out of the plane of the web) of the upright at the point where the diagonal crosses it. The upright is therefore subjected to a distributed transverse restoring load that is proportional to the deflection.

- Fig 6(b) gives the graph for the calculations for double uprights showing the ratio $\mathbf{P_U/P_{UE}}$ as a function of the ratio **d/h**, where P_U is the buckling load of the upright and P_{UE} is the Euler load.

- The assumption of clamped edges would be justified only if the ends of the uprights were fastened rigidly to the flanges and if the flanges had an infinite torsional stiffness. Usually, beam flanges have a rather low torsional stiffness and thus do not justify the assumption of clamped edges for the uprights.

- Single uprights are, in effect, eccentrically loaded columns, as long as the load is infinitesimal. If the uprights are very closely spaced, the web between the uprights must deflect (on the average) in the same manner as the uprights. Under this condition, the eccentricity is equal to the initial value **e** all along the upright and does not change with increase in load. The upright is therefore designed by the formulas used for an eccentrically loaded compression member with negligible deflection.

- If the uprights were extremely widely spaced, the major portion of the web would remain in its original plane (on the average). Consequently, the compressive loads acting on the uprights would remain in its original plane, and the upright would act as an eccentrically loaded column under vertical loads, except for the modification introduced by the elastic transverse support furnished by the web.